

A non-trivial spectrum for the trivial $\lambda\phi^4$ theory

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It is pointed out that one-component ϕ^4 theory in four dimensions has a non-perturbative sector that can be studied by means of an exact duality transformation of its Ising limit. This duality maps it to a membrane model. As a consequence, the ϕ^4 -theory turns out to have, in the broken symmetry phase, a remarkably rich spectrum of physical states corresponding to membrane excitations. A numerical study of the correlators between dual variables allows to evaluate the masses of the first few states.

1. INTRODUCTION

Although the four-dimensional ϕ^4 theory is known to be most likely a trivial field theory, i.e. its continuum limit is identified with an infrared Gaussian fixed point, it does not mean that it is necessarily useless in describing interactions between elementary particles (for instance Higgs particles): once the renormalized coupling g_r at zero momentum has been fixed, the triviality property tells simply us that the ultraviolet cutoff Λ cannot be pushed to arbitrary high values. For small g_r the upper bound on Λ has the asymptotic expansion

$$\log(\Lambda/m_r) \leq A/g_r + B \log(g_r) + C + \dots \quad (1)$$

where m_r is a renormalized mass. As a consequence, this theory cannot actually reach the continuum limit. Nevertheless, if there is a region where the cutoff is much larger than the physical masses entering into the game, it effectively behaves like a continuum theory and can provide an accurate description of an interacting system. I argue in this talk that the spectrum of this theory necessarily contains, in the broken symmetry phase, besides the ground particle, a possibly infinite tower of physical states. For sufficiently low values of the coupling g_r the ratios among the masses of these states are universal constants. The above statements follow directly from an exact duality transformation of the 4D Ising model, which is a particular limit of a lattice regularization of the ϕ^4 -theory. According to

this transformation, the broken symmetry phase of the 4D Ising model is mapped to a membrane model. It follows that the spectrum of the ϕ^4 -theory in such a phase contains an infinite tower of physical states corresponding to membrane excitations. If the system is sufficiently close to the critical point (i.e. g_r sufficiently small) the masses of the physical states have identical scaling behaviours, hence their ratios are expected to be universal. Note that these properties of the spectrum of ϕ^4 theory are not specific of $D = 4$ dimensions : The low temperature phase $T < T_c$ of the Ising model for any $D \geq 3$ has a dual description in terms of extended objects of dimensions $p = D - 2$ which yield a (possibly) infinite set of physical states. For instance, the 3D Ising model is dual, for $T < T_c$, to a string (i.e. $p = 1$) model which describes the confining phase of the \mathbb{Z}_2 gauge theory. The glue-ball spectrum of this theory has been studied only recently [3], showing the existence of a large set of physical states, as expected. In this work we do a similar analysis for the 4D Ising model. As a result, the first few membrane states are detected.

The remaining challenge is to understand how this wide physical spectrum of ϕ^4 -theory at $g_r \neq 0$ can be reconciled with the single particle state of the free Gaussian theory describing its continuum limit for $D \geq 4$. As a possible way out we observe that the excited states of the spectrum are not directly coupled to the local fields ϕ 's (“order operators”; we shall use this notion in this broad sense), but rather to the disorder oper-

ators [1]. These can be written down as non-local and non-polynomial expressions of the ϕ 's (see eq.(8)), while non-perturbative proofs of triviality are based on inequalities among vacuum expectation values of *polynomials* of the order operators [2]. Perhaps the non-perturbative part of the spectrum could be related to the observed non trivial directions in scalar theories with non polynomial potentials [4].

2. THE MODEL

The action of the 1-component ϕ^4 -theory on the (hyper)cubic lattice $\mathcal{L} = \mathbb{Z}^4$ may be parameterized as

$$S_\phi = \beta S_{links} + \sum_{x \in \mathcal{L}} (\phi_x^2 + \lambda(\phi_x^2 - 1)^2) , \quad (2)$$

with

$$S_{links} = - \sum_{x \in \mathcal{L}} \sum_{\mu=1}^4 \phi_x \phi_{x+\hat{\mu}} \equiv - \sum_{links} \phi_{link} \quad (3)$$

Where ϕ is a real field associated to the nodes x of the lattice and $\hat{\mu}$ denotes the unit vector in the μ -direction. Once m_r and g_r have been fixed, the renormalization group trajectories (the “lines of constant physics”) in the plane of the bare parameters flow toward higher values of the bare quartic coupling λ and terminate at the $\lambda \rightarrow \infty$ limit [5], where the action (2) reduces to that of the Ising model (3) with $\phi_x \in \pm 1$. In other terms, the 4D Ising model is the limit theory which saturates the the triviality bound (1): at fixed g_r is the best approximation to the continuum limit .

2.1. Duality transformation

It is well known that the Kramers-Wannier transformation can be extended to the Ising model defined on a lattice of arbitrary dimensions D . In particular, for $D = 4$ the Ising model with action S_{links} admits a dual description in terms of a dual field $\tilde{\varphi}_\square \in \pm 1$ associated to the plaquettes of the dual lattice $\tilde{\mathcal{L}} = (\mathbb{Z} + \frac{1}{2})^4$. The dual action is given by the sum of the contributions of the elementary cubes of $\tilde{\mathcal{L}}$:

$$S_{cubes} = \sum_{cubes \in \tilde{\mathcal{L}}} \tilde{\varphi}_{cube} , \quad \tilde{\varphi}_{cube} = \prod_{\square \in cube} \tilde{\varphi}_\square \quad (4)$$

where the last product runs over the six faces of the cube. The duality transformation states that the partition function $Z_{Ising}(\beta) = \sum_{\phi_x} \exp(-\beta S_{links})$ is proportional to the partition function of the dual description $Z_{dual}(\tilde{\beta}) = \sum_{\tilde{\varphi}_\square} \exp(-\tilde{\beta} S_{cubes})$, with

$$\sinh(2\beta) \sinh(2\tilde{\beta}) = 1 . \quad (5)$$

It is important to stress that this duality transformation is not a symmetry of the model. It maps one description of the dynamical system to another description of the same system. The last equation shows in particular that the low temperature region of the Ising model is mapped into the strong coupling region of the dual model. Such a dual description has a local \mathbb{Z}_2 symmetry generated by any arbitrary function $\eta \in \pm 1$ of the links of $\tilde{\mathcal{L}}$, through the transformation

$$\tilde{\varphi}_\square \rightarrow \tilde{\varphi}_\square \cdot \eta_\square , \quad \eta_\square = \prod_{links \in \square} \eta_{link} . \quad (6)$$

This is the lattice version of the generalized gauge transform of an antisymmetric two-index potential $A_{\mu\nu} \sim \tilde{\varphi}_\square$, i.e. $A_{\mu\nu} \rightarrow A_{\mu\nu} + \partial_\mu \eta_\nu - \partial_\nu \eta_\mu$ which generates the gauge invariant three-index field strength $F_{\mu\nu\rho} \sim \tilde{\varphi}_{cube}$. The local symmetry (6) implies that the disorder observables [1], i.e. the “gauge” invariant quantities of the dual description, are the vacuum expectation values of (products of) surface operators $\tilde{\varphi}_\Sigma$ associated to any arbitrary, closed surface Σ of $\tilde{\mathcal{L}}$:

$$\tilde{\varphi}_\Sigma = \prod_{\square \in \Sigma} \tilde{\varphi}_\square . \quad (7)$$

In the phase corresponding to the broken \mathbb{Z}_2 symmetry of the Ising model, the correlator between surface operators $\langle \tilde{\varphi}_\Sigma \tilde{\varphi}_{\Sigma'} \rangle_{dual}$ has a strong coupling expansion which can be expressed as the sum of weighted 3D random manifolds having Σ and Σ' as boundary. In other terms, the dual of the 4D Ising model is a membrane theory, in the same sense as the strong coupling expansion of any lattice gauge theory can be seen as a string theory, formulated as a sum of weighted random surfaces with Wilson loops as boundary. Using the duality map we can study the membrane physics directly in the Ising model by expressing the disorder observable in terms of the

local field ϕ_x : For a given closed, not necessarily connected surface $\Sigma \subset \tilde{\mathcal{L}}$ choose a 3D manifold $M \subset \mathcal{L}$ having Σ as boundary: $\partial M = \Sigma$. Denote by $\{\text{links} \perp M\}$ the set of links orthogonal to M . One has

$$\langle \tilde{\varphi}_\Sigma \rangle_{dual} = \left\langle \prod_{\{\text{links} \perp M\}} e^{-2\beta\phi_{link}} \right\rangle_{Ising} \quad (8)$$

2.2. The membrane spectrum

In order to study the physical spectrum of the membrane one has to define suitable correlators between surface operators with a recipe very similar to the one used for the glue-ball spectrum of gauge models: *i*) choose a set of n closed surfaces $\{\Sigma_1, \Sigma_2, \dots, \Sigma_n\}$ belonging to a given 3D slice $\mathcal{S}(x_4)$ of the dual lattice, where x_4 plays the role of “time” coordinate; *ii*) evaluate the correlation matrix

$$c_{ij}(t) = \left\langle \sum_{x \in \mathcal{S}(x_4)} \tilde{\varphi}_{\Sigma_i} \sum_{x \in \mathcal{S}(x_4+t)} \tilde{\varphi}_{\Sigma_j} \right\rangle_{dual}; \quad (9)$$

iii) project on states of definite spin and parity.

The sum over x in Eq.(9) is a shorthand notation to denote translational invariant combinations of surface operators. For sufficiently large t the eigenvalues λ_i of $c(t)$ have the asymptotic form $\lambda_i \sim c_i e^{-m_i t}$, where m_i denotes the mass of the i^{th} eigenstate.

3. NUMERICAL RESULTS

Taking advantage of eq.(8), we have evaluated the masses of the lowest membrane states by studying the 4D Ising model with a non-local cluster updating algorithm [6] at $\beta = 0.154$ on a lattice of size $12^3 \times 16$. Here, in a large scale numerical simulation [7], it was observed small finite volume effects and a good agreement with the 3-loop β -function.

We made three different numerical experiments for a total of 3×10^6 iterations, using correlators of disorder operators associated to closed surfaces with the topology of the sphere and of the torus. We chose 12 different shapes with area ranging from 38 to 134 plaquettes and analyzed the channels $J^P = 0^+, 1^-, 2^+$. In the channel $J^P = 1^-$ the signal was too low to extract a reliable quantitative information. Two states have been detected

in the channel $J^P = 0^+$ and one in the channel $J^P = 2^+$. Their masses, in lattice units, are given by

$$m_{0+} = 0.5522(35) , \quad (10)$$

$$m'_{0+} = 1.34(10) , \quad (11)$$

$$m_{2+} = 1.74(22) . \quad (12)$$

As a consistency check, note that the mass of the lowest membrane state m_{0+} should coincide with the physical mass m of the ground particle of the ϕ^4 -theory, because the mass gap of the theory cannot depend on the kind of description employed (the Ising model or its dual). Actually in ref.[7] from analysis of the order-order correlator on a lattice of the same size the value $m = 0.553(3)$ has been reported .

The fact that the lowest particle can be described as a membrane state implies that it couples both to the order operator ϕ_x and to the disorder one $\tilde{\varphi}_\Sigma$. On the contrary the excited membrane states are expected to be decoupled from ϕ_x , otherwise they should manifest themselves by unitarity also in the order-order correlators, where no signal of this sort has been previously reported. Also this prediction is well supported by our analysis.

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